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LOCALIZED EXPLOSION IN A MATERIAL WITH A MAGNETIC FIELD AND THE CONSEQUENCES OF FINITE CONDUCTIVITY IN A MAGNETOHYDRODYNAMIC MODEL

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Introduction. An explosion in an empty space or a rarified gas in the presence of a magnetic field is the prototype of a number of natural cosmic and laboratory processes [1]; experimental explosions in the upper atmosphere [2, 3] have produced a stream of numerical and theoretical works. Magnetic retardation and conversion of plasma cloud energy with dispersion in an empty space has been considered in [4, 5]; in [5] this was done by numerical solution of two-dimensional gas dynamic equations. On the basis of a hybrid model a study was made of collisionless interaction with a magnetized material of unidimensional cylindrical [6] and two-dimensional "spherical" [7] plasma clouds. A unidimensional cylindrical explosion was computed in a magnetohydrodynamic approximation in [8].

Even without a magnetic effect a large scale explosion at a height is two dimensional due to the nonuniformity of the atmosphere over the vertical; detailed calculations are given in [9]. With the action of a magnetic field inclined to the vertical, flow becomes three-dimensional. Naturally, there is an increase in the difficulty of the calculation, and at the highest level the difficulty is aggravated for selecting a physical model (collision—collisionless flow, variability of ionization, etc.). Therefore, in order to understand these phenomena solutions for simple model problems which take account of some part of the actual features of the process are useful. For this purpose in the present work the following step is made compared with [8]: a "spherical" explosion is considered in an MHD-approximation. By means of appropriate averaging with respect to angles the two-dimensional problem in the case of a uniform material is converted to a unidimensional problem. Within the scope of sector [9] approximation the case is studied of a nonuniform atmosphere. In order to solve these problems a second order of accuracy scheme is used for the method of large particles with introduction of artificial viscosity. In conclusion the question is touched upon of refining the approximation of ideal conductivity and the conclusions which emerge as a result of this.

Approximate Reduction of the Two-Dimensional MHD-Problem to a Spherically Symmetrical Problem. We turn to gas dynamic description of motion without discussing the question here of its justification under specific conditions. When concerning this description there are no unconditional contradictions, even natural ionization of a material is sufficient so that the conductivity is assumed to be infinite. Then a magnetic field $\mathbf{H}$ in a gas moving with

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velocity **u** satisfies the equations

$$\partial \mathbf{H}/\partial t = \text{rot}\,[\mathbf{u} \times \mathbf{H}],\ \text{div}\,\mathbf{H} = 0. \tag{1}$$

We shall use spherical coordinates r, θ, reading angle θ from the direction of the undisturbed magnetic field $\mathbf{H}_0$, which is uniform at infinity. We assume that the material is uniform so that $\mathbf{H}_0$ is the axis of symmetry. By assuming that flow is approximately spherically-symmetrical, i.e., by ignoring overflow of a substance through an angle, we write gas dynamic equations:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \rho u = 0, \quad u \equiv u_r; \tag{2}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial r} + \bar{f}_r; \tag{3}$$

$$\frac{\partial}{\partial t}\rho\left(\varepsilon + \frac{u^2}{2}\right) + \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \rho u\left(\varepsilon + \frac{p}{\rho} + \frac{u^2}{2}\right) = \bar{f}_r u. \tag{4}$$

Specific internal energy ε is expressed in terms of pressure p and density ρ: $\varepsilon = p/(\gamma - 1)\rho$ (γ is adiabatic index). The radial component of the pondermotor force

$$f_r = \frac{1}{4\pi}[\text{rot}\,\mathbf{H} \times \mathbf{H}]_r = \frac{H_\theta}{4\pi r}\left[\frac{\partial H_r}{\partial \theta} - \frac{\partial}{\partial r}(rH_\theta)\right] \tag{5}$$

is averaged for angles in Eqs. (3) and (4). If we are interested in movement in a given direction θ on the basis of sector approximation, instead of f_r we should substitute $f_r(\theta)$. In this case the density of the undisturbed material ρ_0 is assumed to be dependent on r, which we consider as nonuniformity of the atmosphere.

In the approximation adopted $\mathbf{u} \equiv \mathbf{u}_r$ does not depend on θ, and Eqs. (1), written in coordinate form

$$\frac{\partial H_r}{\partial r} + \frac{u}{r^2}\frac{\partial}{\partial r}(r^2 H_r) = 0, \quad \frac{\partial H_\theta}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(ruH_\theta) = 0; \tag{6}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 H_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(H_\theta \sin\theta) = 0, \tag{7}$$

permit separation of the variables. Bearing in mind that at infinity $H(r, \theta) = H_0$, we assume that

$$H_r = H_r^0(t, r)\cos\theta, \quad H_\theta = H_\theta^0(t, r)\sin\theta. \tag{8}$$

New functions H_r^0, H_θ^0 obey the same Eqs. (6). According to (7) they are connected by the relationship

$$H_\theta^0 = -\frac{1}{2r}\frac{\partial}{\partial r}(r^2 H_r^0). \tag{9}$$

This equality is useful for transformations as in (7) there is not a variable, which follows from (1) and the identity div rot ≡ 0. In order that it is automatically fulfilled the field prescribed in the initial condition should satisfy the equation div **H** = 0.

By substituting (8) in (5) we see that the radial force which causes magnetic retardation diminishes as $f_r \sim \sin^2\theta$ from the equatorial direction $\theta = \pi/2$ to zero in polar directions θ = 0, π. In fact, this leads to deformation at the beginning of a spherical explosive wave, and to formation of "constrictions" at the surface of the front drawn out along $\mathbf{H}_0$. Taking account of (8), (9), and $\overline{\sin^2\theta} = 2/3$ we present the force in Eqs. (3) and (4) in the form*

$$\bar{f}_r = -\frac{2}{3}\frac{H_\theta^0}{4\pi}\frac{\partial}{\partial r}\left(H_\theta^0 - \frac{H_r^0}{2}\right) = -\frac{2}{3}\frac{\partial}{\partial r}\frac{H_\theta^{02}}{8\pi} + \frac{2}{3}\frac{H_\theta^0}{8\pi}\frac{\partial H_r^0}{\partial r}. \tag{10}$$

*The first term which relates to the action of component H perpendicular to the movement direction may be treated as the action which is averaged with respect to angles for magnetic pressure $\overline{H_\theta^2}/8\pi = (2/3)\, H_\theta^{02}/8\pi$.

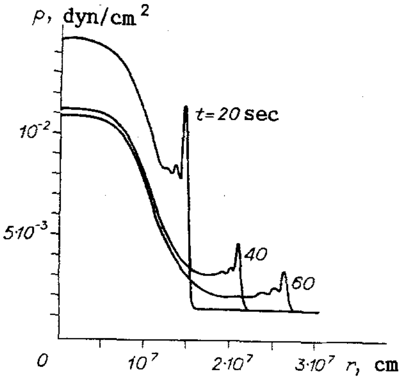

Fig. 1

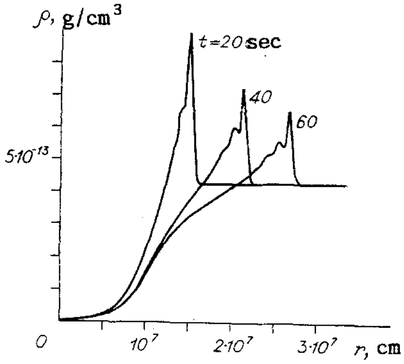

Fig. 2

Equations (2)-(4), (6), and (10) for $H_\theta{}^0$, $H_r{}^0$ form a closed system. Undistrubed parameters for the material ρ_0, p_0, H_0, and explosive energy $\mathscr{E}$ are prescribed.

<u>Computational Method.</u> Calculations of magnetic gas dynamics equations were carried out by the continuous computation method for large particles [10] in a uniform grid with the number of nodes 150-300. According to radial distribution of $H_r{}^0$ and $H_\theta{}^0$ known at this instant force $\bar{f}_r$ is found and Eqs. (2)-(4) are resolved. Then from the distribution u(r) obtained by means of (6) the fields are worked out and the force in the next instant is found from (10), etc. The difference scheme exhibited the property of conservativeness with respect to total energy (including also magnetic energy) and it had a second order of accuracy. Difference methods were studied for introducing an artificial calculation viscosity [11] which provided stability for the solution. Typical values of artificial viscosity coefficients (in the notations of [11]) are: $Q = 0.3$, $\lambda = 1$, $\delta = 0.2$. In selecting the steps with respect to time it did not appear possible to consider the central zone where sound velocity is very high. Here no such instabilities arose. The Courant number was taken as 0.5-0.6. The computation procedure was proved by solving a known problem for explosion in a material with a counterpressure (gas). With good accuracy the results coincide with those provided in [9].

<u>Results.</u> Calculation for a uniform atmosphere is made for the same collection of parameters ($\mathscr{E} = 3\cdot10^{20}$ erg, $H_0 = 0.5$ Oe, $\rho_0 = 4.3\cdot10^{-13}$ g/cm³, $p_0 = 1.8\cdot10^{-3}$ dyn/cm², $\gamma = 5/3$),* which are adopted (partly) in [5] and to which the cylindrical MHD-model [8] is orientated. With these parameters the maximum with respect to angle θ magnetic 'counterpressure' $p_M = H_0{}^2/8\pi = 10^{-2}$ dyn/cm² is greater by a factor of 5.5 than gas pressure p_0, which points to the preferential magnetic mechanism of retardation of a substance in the stage of shock-wave (SW) generation.

Given in Figs. 1-5 are the calculated results: the distributions of p, ρ, u, $H_r{}^0$, $H_\theta{}^0$ with respect to radius at different instants of time t, and also the form of magnetic force lines at instant t = 0.1 sec.† In the earlier stages the distribution of p, ρ, u does not differ from the self-modeling distributions for the problem of a strong localized explosion. In the later stages the SW gradually degenerates into a weak fading disturbance which propagates with a velocity close to the Alfenov velocity $(H_0{}^2/4\pi\rho_0)^{1/2} = 1.5\cdot10^5$ cm/sec. Movement behind the wave gradually ceases. The final pressure leveled out in space tends towards $p_M > p_0$ in contrast to the nonmagnetic case when with $t \to \infty$ pressure returns to atmospheric p_0. In accordance with the equation $\rho d(\mathbf{H}/\rho)/dt = (\mathbf{H}\nabla)\mathbf{u}$, which emerges from (1) and (2), the magnetic field component H_θ evolves, being proportional to gas density ρ (Figs. 2 and 4).

Towards the end of the process approximately 25% of the explosive energy is converted into magnetic energy. Due to irreversible heating of the gas the SW in a central "empty" sphere with $R \approx 1.2\cdot10^7$ cm remains about 30% $\mathscr{E}$. The rest of the energy is carried away with the fading, but embracing all of a large spherical layer, wave.

*This γ relates to the assumption of a considerable degree of dissociation of molecular gas and absence of any reactions due to the rareness of collisions. For the purposes of this work there is no selection of values of γ.

†In Fig. 4 $H_r{}^0 > 0$, $H_\theta{}^0 < 0$, and in Fig. 5 polar axis z is directed along $\mathbf{H}_0$.

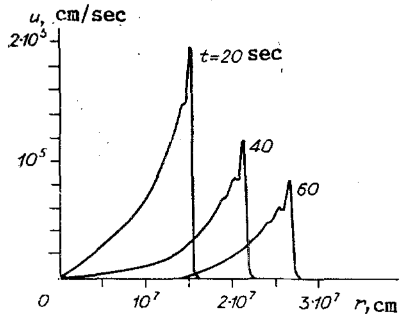

Fig. 3

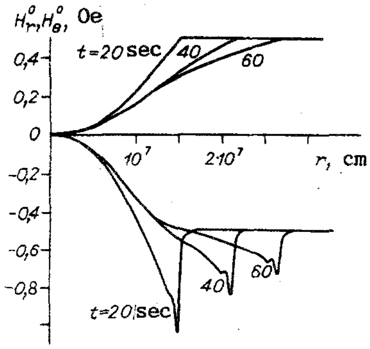

Fig. 4

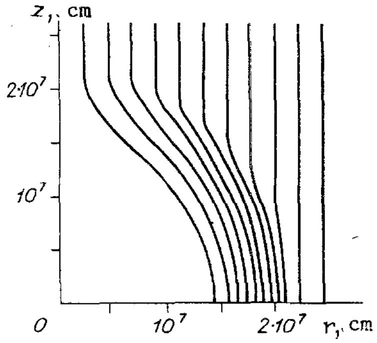

Fig. 5

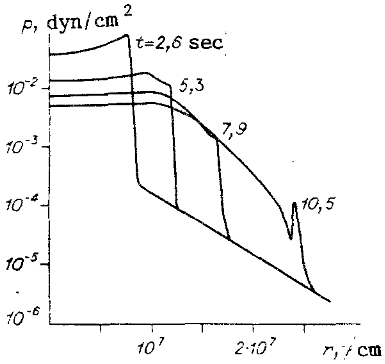

Fig. 6

Shown in Fig. 6 are the results of the case of an exponentially decreasing density $\rho_{00} = \rho_0 \exp(-r/\Delta)$ with $\Delta = 4.2 \cdot 10^6$ cm and the same value of ρ_0 at the point of the explosion. The solution describes propagation of an explosive wave upward with the slope of H_0 to the vertical of approximately $\sqrt{2/3} \approx 54°$. It can be seen that the SW front first slows down in accordance with the general pressure drop in the wave, and then it starts to accelerate since it spreads through a more and more rarified medium.

As with absence of a magnetic field, the wave front recedes to infinity in a finite time. However, the reason for unlimited wave acceleration is now different. In the absence of a field the SW accelerates due to an increase in its amplitude, i.e., the pressure ratio in the front. There is a kind of accumulation of energy behind the SW front. In contrast, in the absence of a constant magnetic counterpressure the SW amplitude falls and the wave gradually degenerates into a weak disturbance. The propagation rate of the disturbance itself increases, which equals the Alfenov velocity $v_A = (H_0{}^2/4\pi\rho)^{1/2} \to \infty$ since magnetic pressure remains unchanged and gas density decreases exponentially.

For a number of reasons it was not possible to prolong computation to long times, and the question of the so-called "breakthrough" of the atmosphere under real conditions when there is a magnetic field remains open. As is well known, in the absence of a magnetic field due to an exponential tendency of pressure towards zero there is no force which could maintain the flow of gas upwards. In principle a magnetic field provides this force. The question becomes thus: whether after all mechanical equilibrium* is established in which immobile gas will be distributed over the height so that the sum of gas and magnetic pressures taking account of the freezing nature of the field in the substance becomes constant over the height. In the future we hope to take this problem to the end.

<u>Nondissipative Effects Caused by Finite Conductivity of the Medium.</u> The idea has been implanted that refinement of MHD-equations for an ideally conducting medium connected with the finite nature of conductivity consists simply of calculating dissipative effects: diffusion of the magnetic field in the plasma, and the release of the Joule heat of flows. In

*Naturally, temporary equilibrium, while the force of gravity, thermal conductivity, etc., do not come into action.

the problem (of the type considered above) these corrections do not add anything qualitatively new and they are very small [8]. We turn attention to another effect of nonideal conductivity leading maybe not necessarily to large effects, but of a qualitative character which within the scope of the MHD-model have not apparently been considered.

Plasma conductivity is $\sigma = e^2n/m\nu_m$ (n is density of electrons, ν_m is effective frequency of their collisions with atoms and ions). Idealization of $\sigma = \infty$ corresponds to the limit $\nu_m \to 0$ or $n \to \infty$. Although disregard for dissipation is equivalent to the first, all the same the MHD-model of an ideally-conducting medium relates to the second. The current density in electrically neutral plasma is

$$\mathbf{j} = en(\mathbf{u} - \mathbf{v}) = (c/4\pi)\,\mathrm{rot}\,\mathbf{H}, \tag{11}$$

where $\mathbf{v}$ is the average, i.e., hydrodynamic, electron gas velocity (as previously the displacement current is not considered). In the equation for movement of an electron gas

$$nm\dot{\mathbf{v}} = -en(\mathbf{E} + c^{-1}[\mathbf{v} \times \mathbf{H}]) + nm\nu_m(\mathbf{u} - \mathbf{v}) - \nabla p_e \approx 0, \tag{12}$$

and as always we disregard the inertia term. This is correct due to the fact that under the action of large Coulomb forces which provide electrical neutrality for the plasma, $\dot{\mathbf{v}} \approx \dot{\mathbf{u}}$, and the mass of an atom $M \gg m$. By combining (12) with an equation of motion for a heavy particle gas which contains similar terms for electric and Lorentz forces and an exchange of electrons with a pulse, we arrive at the previous equation for gas motion (3) in which p is the total pressure of heavy particles and electrons, and in accordance with (11) the pondermotor force has the previous form $\mathbf{f} = c^{-1}[\mathbf{j} \times \mathbf{H}]$. By omitting the not very important term ∇p_e, which describes the diffusion current, we obtain from (11) and (12) Ohm's law

$$\mathbf{j} \approx \sigma(\mathbf{E} + c^{-1}[\mathbf{v} \times \mathbf{H}]),\ \mathbf{E} = -c^{-1}[\mathbf{u} \times \mathbf{H}] + [\mathbf{j} \times \mathbf{H}]/enc^{+}\mathbf{j}/\sigma.$$

By substituting E in the equation for electromagnetic induction we find the final well-known equation for $\mathbf{H}$

$$\frac{\partial \mathbf{H}}{\partial t} = \mathrm{rot}\,[\mathbf{u} \times \mathbf{H}] - \frac{c}{4\pi e}\,\mathrm{rot}\,\frac{1}{n}[\mathrm{rot}\,\mathbf{H} \times \mathbf{H}] - \mathrm{rot}\,\frac{c^2}{4\pi\sigma}\,\mathrm{rot}\,\mathbf{H}, \tag{13}$$

which generalizes (1) in the case of ideal conductivity. Equation (13) is converted into (1) not simply with $\sigma \to \infty$, but only with $n \to \infty$, and without reference to the assumption about ν_m, whether collisions are rare or not. In fact, according to (11) the limit $n \to \infty$ corresponds to "indistinguishability" of $\mathbf{v}$ and $\mathbf{u}$, and the characteristic equality $\mathbf{E} = c^{-1}\cdot[\mathbf{u} \times \mathbf{H}]$ for a moving superconductor.

With finite conductivity in the right-hand part of (13) apart from the third term, part of which describes diffusion permeability of the field towards plasma, the second term (we call it N) appears not to be connected with collision and dissipative processes. For a plane or unidimensional cylindrical MHD-flow, when vector $\mathbf{H}$ is perpendicular to the coordinate on which it depends, $N \equiv 0$. In the two-dimensional (and naturally the three-dimensional) case, i.e., with a localized explosion, $N \neq 0$. In order of value N is $\beta \equiv \Omega_e/\nu_m \equiv eH/mc\nu_m$ times greater than the third "diffusion" term in (13) (Ω_e is cyclotron frequency, β is the so-called Hall parameter).

For the version computed above $\beta \sim 10^4$. However, compared with the main (first) term, which relates to the model of an ideally conducting medium, the value of N may also be small. The ratio of N to the first term is of the order $\delta_1 = cH/4\pi enuL$ (L is scale of length in which the field changes). For our parameters if the gas is considered once ionized, the ratio $\delta_1 \sim 10^{-3}$, but with incomplete ionization it may appear to be marked. The question of the degree of ionization α, and consequently about the qualitative side of the work, requires a special study of the mechanisms and kinetics of ionization which is outside the scope of this work. However, we emphasize that this is in general a question about the ratio of N and the main term, and not about the ratio of N and the diffusion term. Values of ν_m and β are not excessively sensitive to the degree of ionization. The frequency of collisions depends on α not through the density of the disperser (atoms or ions), but only through the dispersion cross section (gas kinetic or Coulomb).

We consider qualitatively the effects which arise due to considering the finiteness of n. This may be done on the basis of the solution obtained above with $n = \infty$ by taking it as a zero approximation and substituting correction terms in which n^{-1} figures. Vector $\mathbf{N}$ has a component N_φ, which leads to generation of an azimuthal magnetic field H_φ. By substituting (8) in N_φ we find that $H_\varphi \sim \sin\theta \cos\theta$. The azimuthal field is absent at the poles and at the equator it is a maximum at angles of 45 and 135° to the polar axis of $\mathbf{H}_0$. This result was also obtained in [7] within the scope of a hybrid model where naturally the same equalities (11) and (13) figure.

If in the model with n, $\sigma = \infty$ there is only azimuthal current j_φ, then in the following approximation the components j_r and j_θ appear. Lines for an additional current form in meridional planes φ = const of a system of concentric contours, one in each of the four quadrants. In space current lines form, as it were, two sets of planes enclosing each other; one above and the other below the equator. Interaction of current j_r with field H_θ and current j_θ with field H_r leads to development of an azimuthal component for pondermotor force f_φ which twists the plasma expanding from the center around polar axis $\mathbf{H}_0$. The twisting force is symmetrical with respect to the plane of the equator and it tends towards zero at the poles. Part of the explosive energy is converted into rotary motion.

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